Form F. 1624.

THE ENGLISH ELECTRIC COMPANY LIMITED

DEPT.: MECHANICAL ENGINEERING LABORATORY, WHETSTONE.

REPORT No. W/M(4B).p.1023
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Date 7.1.65

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SUMMARIES

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SOLUTION OF FINITE-DIFFERENCE EQUATIONS BY SUMMARY REPRESENTATION

(Investigation No. 10072)

Report by

G.J. TEE

SUMMARY

The technique of "summary representation", which has been developed by G.N. Polozhii for the solution of systems of finite difference equations, istested by applying it to the simple case of the five-node Laplace operator over a rectangle, with Dirichlet boundary conditions. The program solves the problem rapidly and with high accuracy.

MECHANICAL ENGINEERING LABORATORY
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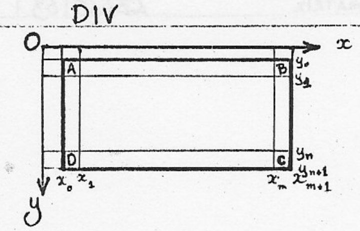
Date 1964
Sheet No. 1, Appendix I

Solution of Dirichlet Problem for a Rectangular Net by Polozhii's Technique.

GIP8(with triads of codes at tracks 129 & 130, & "1" as a 1×1 binary matrix at track 128), N=22, LR07B/2 (1), LR16BT (2), LP15BT (3), LZ34B/1 (4), LZ12B/2 (5), LZ19B/1 (6), LZ40B (7), LZ46B (8), LZ61BM/2 (9,10), LZ51B (11), LZ48B (12), LZ18B/1 (13), LZ66BM (14), LZ58B (15), LZ24B (16), LM12B (17), LZ63BM/1 (18), LM08B (19,20,21), ZC12B (22); 2 TRIADS; u_0^T, u_{m+1}^T, v_0, v_{n+1}. ($m \le 28$, $n \le 28$).

CARD Nos.	Code No.	a	b	c	r	Punch
READ SECOND TRIAD INTO THIRD POSITION	0	0	1	2	47	2
80 → MOVE TRIAD FROM 130 TO SECOND POSITION, ZC12B	1	130	1	250	22	3
READ u_0^T ($1\times n$) b=1 (u on AD) LR07B/2	2	0	1	131	1	4
READ u_{m+1}^T ($1\times n$) b=2 (u on BC) "	3	0	2	132	1	5
READ v_0 ($1\times m$) b=3 (u on AB) "	4	0	3	133	1	6
READ v_{n+1} ($1\times m$) b=4 (u on DC) "	5	0	4	134	1	7
85 → FIND n LZ34B/1	6	131	0	0	4	8
STORE nP9	7	107	0	95	40	9
FETCH "1" (1×1) LZ12B/2	8	128	0	0	5	Y
EXPAND AS	9	95	10	11	45	X
ROW VECTOR ($1\times n$) LZ19B/1	10	1	0	0	6	0
REVERSE SUM [n,⋯,2,1] LZ40B	11	0	0	1	7	1
EXTRACT	12	1	0	1	48	2
"n" (1×1) LZ46B	13	0	1	0	8	3
ADD	14	1	0	0	48	4
"n+1" (1×1) LZ61BM/2	15	128	0	135	9	5
SUB	16	2	0	0	48	6
[1,2,⋯, n] LZ61BM/2	17	135	1	136	9	7
[1,2,⋯, n] "	18	135	1	0	9	8
CHANGE TO COLUMN VECTOR ($n\times 1$) LZ51B	19	0	1	0	11	9
NULL ($n\times n$) LZ24B	20	0	136	1	16	Y
ADJUST b.p. OF NULL LM12B	21	136	0	1	17	X
DIV	22	4	0	0	48	0
COLUMN OF $\left(\frac{1}{n+1}, \cdots, \frac{n}{n+1}\right)$ LZ61BM/2	23	0	135	137	9	1
DOUBLE IT LZ51B	24	137	0	255	11	2
ADD	25	1	0	0	48	3
n ROWS OF $\frac{1,2,\cdots,n}{1,n,\cdots,n}$ LZ61BM/2	26	136	1	32	9	4
MULT	27	3	0	0	48	5
MATRIX OF $\frac{2ik}{n+1}$ ($n\times n$) LZ61BM/2	28	137	32	0	9	6
$\sqrt{\frac{1}{2}(n+1)}\,P$ ($n\times n$) LZ48B	29	0	1	96	12	7
($p_{ik} = \sqrt{\frac{2}{n+1}} \sin \frac{ik\pi}{n+1}$).	30					8
DIV	31	4	0	0	48	9



MECHANICAL ENGINEERING LABORATORY
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Date
Sheet No. 2, Appendix I

Track 130

$$\eta_k = 2 - \cos\frac{k\pi}{n+1}, \quad \nu_k = \eta_k - \sqrt{\eta_k^2 - 1}$$

CARD Nos.		Code No.	a	b	c	r	Punch
$\frac{1}{n+1} u_0^T$ $(1\times n)$	LZ61BM/2	32	131	135	4	9	2
$\frac{1}{n+1} u_{m+1}^T$ $(1\times n)$	"	33	132	135	5	9	3
MATRIX MULT. $\frac{1}{n+1} u_0^T P \sqrt{\frac{1}{2}(n+1)} = \frac{1}{\sqrt{2n+2}} \tilde{u}_0^T$	LM08B	34	4	96	131	19	4
" " $\frac{1}{\sqrt{2n+2}} \tilde{u}_{m+1}^T$	"	35	5	96	132	19	5
$\lambda_k = \cos\frac{k\pi}{n+1}$ $(n\times 1)$	LZ18B	36	137	0	0	12	6
SUB		37	2	0	0	48	7
$\lambda_k - 1$ $(n\times 1)$	LZ61BM/2	38	0	128	1	9	8
$\eta_k = 2 - \lambda_k$ $(n\times 1)$	"	39	128	1	0	9	9
MULT		40	3	0	0	48	Y
η_k^2 $(n\times 1)$	LZ61BM/2	41	0	0	2	9	X
SUB		42	2	0	0	48	0
$\eta_k^2 - 1$ $(n\times 1)$	LZ61BM/2	43	2	128	1	9	1
$\sqrt{\eta_k^2 - 1}$ $(n\times 1)$	LZ18B/1	44	1	0	2	13	2
SUB		45	2	0	0	48	3
$\nu_k = \eta_k - \sqrt{\eta_k^2 - 1}$ $(n\times 1)$	LZ61BM/2	46	0	2	127	9	4
ν_k $(1\times n)$	LZ51B	47	127	0	0	11	5
FLOAT	LZ66BM	48	127	0	0	14	6
$\log_e \nu_k$ $(1\times n)$	LZ58BM	49	0	0	126	15	7
CHANGE v_0 TO COL. $(m\times 1)$	LZ51B	50	133	1	0	11	8
NULL $(m\times n)$	LZ24B	51	133	132	0	16	9
CHANGE v_0^T BACK TO ROW $(1\times m)$	LZ51B	52	133	0	0	11	Y
ADJUST b.p.	LM12B	53	126	0	0	17	X
ADD		54	1	0	0	48	0
m ROWS OF $\log_e \nu_k$ $(m\times n)$	LZ61BM/2	55	126	0	32	9	1
FIND m	LZ34B/1	56	133	0	0	4	2
STORE IN 94, AS mP9		57	107	0	94	40	3
FETCH "1" (1×1)	LZ12B/2	58	128	0	0	5	4
EXPAND "1" AS $1\times m$ VECTOR		59	94	10	60	45	5
REVERSE SUM $[m,\dots,2,1]$	LZ40B	60	0	0	138	7	6
CHANGE TO COL. $(m\times 1)$	LZ51B	61	138	1	0	11	7
EXTRACT "m"		62	138	0	1	48	8
AS 1×1 MATRIX	LZ46B	63	0	1	0	8	9

Date

Sheet No. 3, Appendix I.

Second Triad of Codes.

(Third Position)

$$u_i = P[I-N^{2(m+1)}]^{-1}\left[N^i(I-N^{2(m+1-i)})Pu_0 + N^{m+1-i}(I-N^{2i})Pu_{m+1}\right]$$

or, $$u_i = P(D_i Pu_0 + D_{m+1-i} Pu_{m+1}).$$

or, $$u_i = P\tilde{u}_i, \quad \text{where } \tilde{u}_i = D_i \tilde{u}_0 + D_{m+1-i}\tilde{u}_{m+1}.$$

CARD Nos.		Code No.	a	b	c	r	Punch
MOVE TRIAD FROM 129 TO SECOND POSITION	ZC12B	64	129	1	250	22	2
JUMP TO 32′		65	0	0	32	33	3
63′ → ADD		66	1	0	0	48	4
$(2n+2)^{-\frac{1}{2}}\tilde{u}^T$ $(m\times n)$	LZ61BM/2	67	32	64	64	9	5
$\sqrt{\frac{2}{n+1}}\,\tilde{u}^T$ $(m\times n)$ $(\to 75)$	LZ51B	68	64	0	255	11	6
(FIRST PROBLEM) MATRIX MULT:- $u^1 = \sqrt{\frac{n+1}{2}}P\sqrt{\frac{2}{n+1}}\,\tilde{u}$ $(n\times m)$	LM08B	69	96	64	0	19	7
PUNCH u^1 $(n\times m)$	LP15BT	70	0	0	33	3	8
INTERCHANGE	ZC12B	71	131	2	0	22	9
u_0^T & u_{m+1}^T WITH	"	72	133	2	131	22	Y
v_0 & v_{n+1}	"	73	0	2	133	22	X
JUMP TO 84		74	0	0	84	33	0
68 → (SECOND PROBLEM) MATRIX MULT:- u^2 $(n\times m)$	LM08B	75	64	96	0	19	1
READ u^1 $(n\times m)$	LR16BT	76	0	0	32	2	2
ADD		77	1	0	0	48	3
$u = u^1 + u^2$ $(n\times m)$	LZ61BM/2	78	0	32	64	9	4
PUNCH u $(n\times m)$	LP15BT	79	64	1	33	3	5
RE-ENTER FOR NEXT CASE		80	0	0	1	33	6
51′ → MULT		81	3	0	0	48	7
$\nu_k^{2(m+1-i)}$ $(m\times n)$	LZ61BM/2	82	0	0	64	9	8
JUMP TO 52′		83	0	0	52	33	9
74 → RESTORE 130 TO SECOND POSITION	ZC12B	84	130	1	250	22	Y
SOLVE SECOND PROBLEM, GIVING u_i^2.		85	6	68	75	46	X
		86					0
		87					1
		88					2
		89					3
		90					4
		91					5
		92					6
		93					7
		94		(m)			8
		95		(n)			9

MECHANICAL ENGINEERING LABORATORY
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Date
Sheet No. 4, Appendix I

Track 129.

$$d_{ki} = \frac{\nu_k^{\,i}\left(1-\nu_k^{\,2(m+1-i)}\right)}{\left(1-\nu_k^{\,2(m+1)}\right)}.$$

$$(k = 1, \dots, n;\ i = 1, \dots, m)$$

CARD Nos.	Code No.	a	b	c	r	Punch
(65)→ ADD	32′	1	0	0	48	2
"m+1" (1×1) LZ61BM/2	33′	128	0	139	9	3
SUB	34′	2	0	0	48	4
COL. OF $(1, 2, \dots, m)$ LZ61BM/2	35′	139	138	140	9	5
MULT	36′	3	0	0	48	6
$i \log_e \nu_k$ $\left(\begin{smallmatrix} k = 1, \dots, n \\ i = 1, \dots, m \end{smallmatrix}\right)$ LZ61BM/2	37′	32	140	0	9	7
$(m+1-i)\log_e \nu_k$ $\left(\begin{smallmatrix} k = 1, \dots, n \\ i = 1, \dots, m \end{smallmatrix}\right)$ "	38′	32	138	64	9	8
EXPONENTIATE : $\nu_k^{\,i}$ " LZ63BM/1	39′	0	0	32	18	9
" $\nu_k^{\,m+1-i}$ " "	40′	64	0	0	18	Y
EXTRACT	41′	0	0	1	48	X
ROW OF $\nu_k^{\,m}$ $(1\times n)$ LZ46B	42′	0	0	64	8	0
MULT	43′	3	0	0	48	1
$\nu_k^{\,m+1}$ $(1\times n)$ LZ61BM/2	44′	64	127	65	9	2
$\nu_k^{\,2(m+1)}$ $(1\times n)$ "	45′	65	65	64	9	3
SUB	46′	2	0	0	48	4
$1-\nu_k^{\,2(m+1)}$ $(1\times n)$ LZ61BM/2	47′	128	64	65	9	5
DIV	48′	4	0	0	48	6
$(2n+2)^{-\frac{1}{2}}\left(1-\nu_k^{\,2(m+1)}\right)^{-1}\tilde{u}_0^{\,T}$ $(1\times n)$ LZ61BM/2	49′	131	65	94	9	7
$(2n+2)^{-\frac{1}{2}}\left(1-\nu_k^{\,2(m+1)}\right)^{-1}\tilde{u}_{m+1}^{\,T}$ $(1\times n)$ "	50′	132	65	95	9	8
JUMP TO 81	51′	0	0	81	33	9
(83)→ SUB	52′	2	0	0	48	Y
$1-\nu_k^{\,2(m+1-i)}$ $(m\times n)$ LZ61BM/2	53′	128	64	64	9	X
MULT	54′	3	0	0	48	0
$\nu_k^{\,i}\left(1-\nu_k^{\,2(m+1-i)}\right)$ $(m\times n)$ LZ61BM/2	55′	32	64	64	9	1
$(2n+2)^{-\frac{1}{2}}\left(1-\nu_k^{\,2m+2}\right)^{-1}\nu_k^{\,i}\left(1-\nu_k^{\,2(m+1-i)}\right)\tilde{u}_0^{\,T}$ "	56′	94	64	64	9	2
$\nu_k^{\,2i}$ $(m\times n)$ "	57′	32	32	32	9	3
SUB	58′	2	0	0	48	4
$1-\nu_k^{\,2i}$ $(m\times n)$ LZ61BM/2	59′	128	32	32	9	5
MULT	60′	3	0	0	48	6
$\nu_k^{\,m+1-i}\left(1-\nu_k^{\,2i}\right)$ $(m\times n)$ LZ61BM/2	61′	32	0	0	9	7
$(2n+2)^{-\frac{1}{2}}\left(1-\nu_k^{\,2m+2}\right)^{-1}\nu_k^{\,m+1-i}\left(1-\nu_k^{\,2i}\right)\tilde{u}_{m+1}^{\,T}$ "	62′	0	95	32	9	8
JUMP TO 66	63′	0	0	66	33	9